Definición de derivada

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} \qquad \boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Ej $f(x) = x^2 + 3$

a) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 3 - (x^2+3)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + h^2 + 2xh \cancel{+3} \cancel{-x^2} \cancel{-3}}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2xh}{h} = \left[\frac{0}{0}\right] = \lim_{h \to 0} \frac{\cancel{h}(h+2x)}{\cancel{h}} = 2x$$

b) $$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{(3+h)^2 + 3 - 12}{h} = \lim_{h \to 0} \frac{\cancel{9} + h^2 + 6h \cancel{+3} \cancel{-12}}{h} =$$

$$= \lim_{h \to 0} \frac{h^2 + 6h}{h} = \left[\frac{0}{0}\right] = \lim_{h \to 0} \frac{\cancel{h}(h+6)}{\cancel{h}} = 6$$

Derivadas de las funciones

Polinómicas

$$\boxed{D(f^n) = n \cdot f^{n-1} \cdot f'}$$

Ejemplos

① $D(x^7) = 7 \cdot x^6 \cdot 1 = 7x^6$

② $D(x^3) = 3x^2$

③ $D((x-3)^4) = 4 \cdot (x-3)^3 \cdot D(x-3) = 4(x-3)^3$

$D(x) - D(3) = 1 - 0$

④ $D(\sqrt{x}) = D(x^{1/2}) = \frac{1}{2} x^{-1/2} \cdot 1 = \frac{1}{2\sqrt{x}}$

⑤ $D\left(\frac{1}{x-3}\right) = D((x-3)^{-1}) = -1 \cdot (x-3)^{-2} \cdot D(x-3) = -\frac{1}{(x-3)^2}$

Logarítmicas

$$\boxed{D(\log_a f) = \frac{f'}{f} \cdot \log_a e}$$

Ejemplos

① $D(\log x) = \frac{1}{x} \cdot \log e$

② $D(\ln x^3) = \frac{3x^2}{x^3} \cdot \cancel{\ln e}$

Exponenciales

$$\boxed{D(a^f) = a^f \cdot f' \cdot \ln a}$$

Ejemplos

① $D(2^x) = 2^x \cdot 1 \cdot \ln 2 = 2^x \cdot \ln 2$

② $D(e^{x^2}) = e^{x^2} \cdot 2x \cdot \cancel{\ln e} = e^{x^2} \cdot 2x$

Derivadas de las operaciones

$D(f+g) = f' + g'$ $\qquad$ $D(3x+5) = D(3x) + D(5)$

$D(f-g) = f' - g'$ $\qquad$ $D(2x^2-5x) = D(2x^2) - D(5x)$

$D(f \cdot g) = f'g + fg'$ $\qquad$ $D(x(x-2)) = D(x)\cdot(x-2) + x \cdot D(x-2)$

$D(f/g) = \dfrac{f'g - fg'}{g^2}$ $\qquad$ $D\left(\dfrac{x+1}{5x}\right) = \dfrac{D(x+1)\cdot 5x - (x+1)\cdot D(5x)}{(5x)^2}$

$D(f \circ g) = f'(g(x)) \cdot g'(x)$ $\qquad$ $D\left[(5x)^3\right] = D(x^3) \cdot 5x \cdot D(5x)$

$f(g(x)) = (5x)^3$

$f = x^3$

$g = 5x$

$$\boxed{\begin{array}{l} D(x) = 1 \\ D(a) = 0 \end{array}}$$

Ejemplos de derivadas

$D(K \cdot f) = K \cdot D(f)$

① $D(x^{12}) = 12 \cdot x^{11} \cdot D(x) = 12 \cdot x^{11}$

$1 - 0$

② $D((x-5)^3) = 3 \cdot (x-5)^2 \cdot D(x-5) = 3(x-5)^2 \cdot \left(D(x) - D(5)\right) = 3(x-5)^2$

③ $D(\sqrt{5x}) = D\left((5x)^{1/2}\right) = \frac{1}{2} \cdot (5x)^{-1/2} \cdot D(5x) = \frac{1}{2\sqrt{5x}} \cdot \left(\underbrace{\cancel{D(5)}}_{0}\cancel{x} + 5 \cdot D(x)\right) =$

$= \frac{1}{2\sqrt{5x}}(5) = \frac{5}{2\sqrt{5x}}$

④ $D\left(\frac{1}{(3x)^2}\right) = D\left((3x)^{-2}\right) = -2(3x)^{-3} \cdot D(3x) = \frac{-2}{(3x)^3} \cdot 3 = \frac{-6}{(3x)^3}$

⑤ $D\left(\log(3x^2)\right) = \frac{D(3x^2)}{3x^2} \cdot \log e = \frac{\cancel{3} D(x^2)}{\cancel{3} x^2} \cdot \log e = \frac{2x}{x^2} \log e = \frac{2 \log e}{x}$

⑥ $D\left(\ln(100x^7)\right) = \frac{D(100x^7)}{100x^7} = \frac{\cancel{100} \cdot D(x^7)}{\cancel{100} x^7} = \frac{7x^6}{x^7} = \frac{7}{x}$

(7) $D(3^{5x-2}) = \underline{3^{5x-2} \cdot D(5x-2) \cdot \ln 3} = 3^{5x-2} \cdot \ln 3 \cdot (D(5x) - D(2)) =$

$= 3^{5x-2} \cdot \ln 3 \cdot 5x$

(8) $D(e^{8x^2}) = e^{8x^2} \cdot D(8x^2) = e^{8x^2} \cdot 16x$

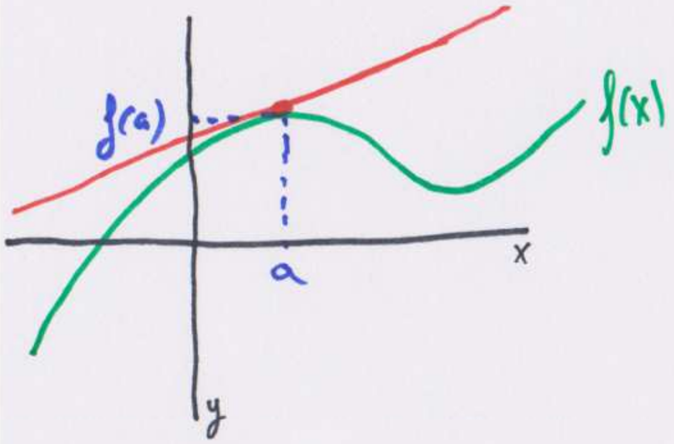
(9) $D(5x^3 + 2x + 3) = 15x^2 + 2$

(10) $D\left(\frac{\ln x}{x^2}\right) = \frac{D(\ln x) \cdot x^2 - \ln x \cdot D(x^2)}{(x^2)^2} = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} =$

$= \frac{x - \ln x \cdot 2x}{x^4} = \frac{x(1 - 2\ln x)}{x^4} = \frac{1 - 2\ln x}{x^3}$

RECTA TANGENTE

A $f(x)$ EN $x=a$



$$\boxed{y - f(a) = \overset{= m \text{ (pendiente)}}{f'(a)} (x-a)}$$

EJEMPLO 1

Calcula la recta tangente a $f(x) = 5x^2 + x$ en $x = 1$.

$a = 1$

$f(a) = 5 + 1 = 6$

$f'(x) = 10x + 1$

$f'(a) = 10 + 1 = 11$

$$y - 6 = 11(x-1)$$

EJEMPLO 2

Halla la recta tangente a $y = x^2 - 5x + 6$ que es paralela a $3x + y - 2 = 0$.

misma pendiente

$y = -3x + 2$

$m = f'(a)$

$f'(x) = 2x - 5$

$f'(a) = -3 \Rightarrow 2a - 5 = -3 \Rightarrow 2a = 2 \Rightarrow a = 1$

$f(1) = 1 - 5 + 6 = 2$

$$y - 2 = -3(x-1)$$